Decomposition of Solid Propellants in a Combustion Chamber

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Abstract

Solid propellants are fuels that do not require an external supply of oxygen for combustion. Once ignited, they rapidly decompose to evolve hot gaseous products. Solid propellants are hence used as fuel for propulsion applications in oxygen rare environments. In this study, a simple model for solid propellant decomposition in a combustion chamber with an exit port was presented. The combustion chamber is assumed to be a 'well-mixed reactor'. The model equations take into account the effect of shape and size of the solid propellant. The phenomenon of 'choking flow' is included calculating the exit flow-rate. Appropriate methodology for calibrating the model parameters was developed. The calibrated model was validated by comparing simulation results with experimental pressure profiles. This study would be useful for providing guidelines to select appropriate solid propellant configurations and for the development of a system level model for propulsion systems.

Keywords

Solid Propellant; Decomposition; Pressure Build-up; Choking Flow

Introduction

Solid propellants are typically used in applications such as rockets and missiles. They are a composite mixture with the separate ingredients such as fuel, oxidizer, binders and plasticizers that macroscopically indistinguishable. Typically propulsion applications, a solid propellant brick is placed inside a closed combustion chamber which has an exit port for the gaseous products. The brick is ignited by the means of an electrical or mechanical shock. Upon ignition the solid propellant brick undergoes self sustained burning leading to rapid evolution of extremely hot gases. This process builds up the pressure in the combustion chamber significantly (~ 100 bar). Gases escape through an exit port due to the built up pressure. These gaseous products are used directly to provide thrust (as in the case of rockets) or alternatively to drive a turbine.

Design and development of such propellant systems often require extensive experimental testing. It is essential to complement such an experimental program by mathematical modeling for enhancing its effectiveness. The actual processes taking place during the decomposition of solid propellant are quite complex and span a wide range of spatio-temporal scales. It is however essential to develop a simple system level model to facilitate interpretation of experimental data and provide guidelines for system design. In this work, we present the development of a simple model for the combustion chamber keeping the requirements of the system level model in mind.

Brief Review of Previous Work

Numerous experimental and computational studies dealing with different aspects of solid propellant combustion have been carried out. In most of these studies, dependence of burning-rate of the solid propellant on pressure is obtained experimentally and the correlations are derived to relate burning rate to various process parameters like chamber pressure, initial solid propellant brick temperature etc. (Clemins et. al., 1975; Panella et. al., 1966; Robert et. al., 1999). The St. Robert's Law is commonly used to correlate propellant burning rate to the chamber pressure (Panella et. al., Robert et. al.). The parameters required by the St. Roberts law are specific to a solid propellant composition and depend on the initial propellant temperature (Wu et. al.).

Deviations from the St. Roberts Law for burning-rate are observed in certain cases due to erosive burning. Erosive burning is the phenomenon of local increase in the burning rate of solid propellant grain due to the cross-flow of gaseous products along its burning surface. The solid propellant bricks are generally internal burning and have a channel (in the form of a cylindrical bore) which allow decomposition products to flow (King, 1978; Kulkarni et. al., 1993; Green, 1954). Erosive burning is typically observed in such solid propellant configurations and the effect is most pronounced in the initial stages of combustion (King, 1978).

Wu et al. (2000) investigated the combustion of solid propellant grains in two-stage air bag inflators used in cars. They assumed that the burning-rate of the solid propellant grain follows an empirical relationship known as the St. Roberts law. Model predictions showed reasonable agreement to experiments performed for two stage air-bag inflators. The model was then used to investigate the effect of various parameters which would be useful for design applications.

Numerous studies have been done that focus specifically on understanding the solid propellant combustion process addressing all the complex reaction schemes occurring during such a process (Beckstead et. al. (2007)). The recent comprehensive models are able to predict not only the burning-rate properties, but also the surface conditions and other key burning characteristics. Various key parameters and underlying processes that dictate the propellant burning and ignition behavior have been identified.

Keeping in mind the aim - to develop a simple model to characterize the process, in the present study we have decided to retain only the essential elements in the mathematical model to ensure simplicity. Erosive burning is a prominent phenomenon especially in internal burning solid propellants. However, this phenomenon will not be considered while modeling the present system. The detailed kinetic reactions and chemical processes were not considered to predict the burning rate. Instead the St. Roberts law was used to model the burning rate of the solid propellant. A justification about why these assumptions are appropriate within the scope of the present study is provided later. Burning occurs at the surface of the solid propellant. As the burning proceeds the solid propellant surface regresses changing the shape and size of the solid propellant thus changing the available

surface area for burning. This change in turn affects the burning rate significantly. Monitoring the burning surface is thus critical in predicting the burning profile of the solid propellant. This aspect is therefore retained in the present model. The model equations and numerical methods to solve these equations are discussed in the following section. An easy way to comply with the journal paper formatting requirements is to use this document as a template and simply type your text into it.

Mathematical Model

Illustrations of the considered combustion chamber and a typical brick of solid propellant are shown in Figure 1. As the solid propellant burns, the internal surface and the surface at the sides regress, changing the shape and size of the solid propellant brick. Tracking the evolution of the solid propellant brick shape is hence crucial to characterize the process. At high pressures, which are expected in this system, the compressibility effects of the gas become important. Furthermore, the velocity of the exiting gas cannot exceed the velocity of sound irrespective of the high pressure differential. This phenomenon is known as 'choking flow' and is expected to occur in our system. The combustion chamber is cooled externally by a cooling coil (not shown in the figure), although the cooling is not significant compared to the rate of evolution of heat due to combustion.

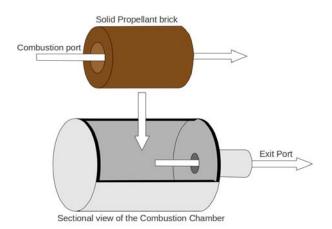


FIG. 1 SCHEMATIC OF COMBUSTION CHAMBER

Based on this physical picture, the following key assumptions were made to simplify the mathematical model without compromising key aspects:

- The combustion chamber is assumed to be a 'perfectly well-mixed'.
- The combustion gases follow the ideal gas law.

- The solid propellant burning follows the St. Robert's burning law. The burning occurs only at the internal and the side surfaces of the solid propellant.
- The outlet flow-rate is modeled as a compressible flow through an adiabatic nozzle. This 'ideal' flow model is then corrected to account for the given exit geometry by a procedure given by Levenspiel (1984).

Based on these assumptions, the processes in occurring in the combustion chamber can be represented by the following set of gas phase mass, energy and species balance equations:

$$\frac{d(\rho_g(V_{ch} - V_{sp}))}{dt} = \dot{m}_{burn} - \dot{m}_{exit} = RHS_{ME}$$
 (1)

$$\frac{d(\rho_g(V_{ch} - V_{sp})h_g)}{dt} = \dot{m}_{burn}(-\Delta H_{R,s}) - h_{eff,sp}(T_g - T_{sp})$$

$$-h_{eff,ch}(T_g - T_{ch}) - \dot{m}_{exit}h_g = RHS_{HE} \quad (2)$$

$$\frac{d\left(\rho_{g}\left(V_{ch}-V_{sp}\right)Y_{m,g}\right)}{dt}=\dot{m}_{burn}Y_{m,s}-\dot{m}_{exit}Y_{m,g}=RHS_{SE} \quad (3)$$

Heat is lost from the decomposition products to the solid propellant brick and the combustion chamber. Heat balances are also formulated for the solid propellant brick and the combustion chamber. These are as follows:

$$\frac{d(m_{sp}C_{p,sp}T_{sp})}{dt} = h_{eff,sp}(T_g - T_{sp})$$
(4)

$$\frac{d\left(m_{ch}C_{p,ch}T_{ch}\right)}{dt} = h_{eff,ch}\left(T_g - T_{ch}\right) - \dot{Q}_{HX}$$
 (5)

Where, heat is removed from the combustion chamber walls using a cooling coil. The rate of removal of heat using the coil is given by:

$$\dot{Q}_{HX} = \left(\dot{m}C_P \left(T_{exit} - T_{inlet}\right)\right)_{coolant} \tag{6}$$

Due to the large coolant flow-rate it was observed experimentally that the coolant exit temperature does not change significantly. Moreover, the magnitude of the rate of heat exchanged is very small in comparison with the magnitude of the rate of heat generated. Hence, in the present study we have assumed the exit temperature of the coolant to be constant. The values of C_{p,sp} and C_{p,ch} are assumed to be constant.

Equations for Solid Propellant Dimensions

The solid propellant is used in this study is in the shape of a hollow cylindrical brick. Burning occurs only at the internal and the side surfaces of the solid propellant brick. The outer surface of the solid propellant brick is purposefully inhibited (i.e. prevented from burning). This is due to the fact that the solid propellant burning rate is dependent on the burning surface area available. The rate of burning of the solid propellant is given by the St. Roberts' law (Wu et. al., 2000):

$$\dot{s} = aP^n \tag{7}$$

Where \dot{s} is the rate of regression of the solid propellant surface. 'a' and 'n' are parameters specific to the solid propellant. Hence,

$$\dot{m}_{burn} = A_b x \rho_{sp} x \dot{s} \tag{8}$$

The burning (regressing surfaces) progressively changes the shape and size of the solid propellant. For this system, the solid propellant brick is cylindrical with a cylindrical bore. Burning occurs only at the inner and the side surfaces. The length regresses from two sides while the inner diameter also increases by twice the rate of regression of the surfaces. The outer diameter (D₀) remains fixed as the outer surface is inhibited. Hence, we obtain two equations for the rate of change of the length (L) and inner diameter (D_i) of the Solid Propellant as follows:

$$\frac{d(L)}{dt} = -2\dot{s} \tag{9}$$

$$\frac{d(D_i)}{dt} = +2\dot{s} \tag{10}$$

From these dimensions, the area for burning and the volume of solid propellant are calculated for each instance as follows:

$$A_b = \pi x D_i x L + \pi x \left(D_o^2 - D_i^2 \right) / 2 \tag{11}$$

$$V_{sp} = \pi x (D_o^2 - D_i^2) x L / 4$$
 (12)

The decomposition terminates when D_i becomes equal to D_0 or when L becomes equal to zero, whichever occurs earlier.

Equations (3), (4) & (5) can be rewritten to obtain differential equations which express the change in the state variables which are of direct interest (P, T & Y_m) as follows:

$$\rho_{g} (V_{ch} - V_{sp}) \frac{dY_{m,g}}{dt} = RHS_{SE} - Y_{m,g} RHS_{ME}$$

$$\rho_{g} (V_{ch} - V_{sp}) C_{p,g} (T) \frac{dT_{g}}{dt} = RHS_{HE} - h_{g} RHS_{ME}$$
 (14)

And By differentiating the ideal gas equation w.r.t. time, we obtain:

$$(V_{ch} - V_{sp}) \frac{dP_g}{dt} = RT_g \frac{dn_g}{dt} + n_g R \frac{dT_g}{dt} - \frac{P_g \dot{m}_s}{\rho_{sp}}$$
 (15)

as
$$\frac{dV_{sp}}{dt} = \frac{-\dot{m}_{burn}}{\rho_{sp}}$$

where from a mole balance we get:

$$\frac{dn_g}{dt} = \frac{\dot{m}_{burn}}{\sum_{m} Y_{m,sp} MW_m} - \frac{\dot{m}_{exit}}{\sum_{m} Y_{m,g} MW_m}$$
(16)

Hence, equations (4), (5), (9), (10), (13), (14) & (15) form our system of ordinary differential equations which describe the processes occurring in the combustion chamber.

Outlet Flowrate Calculation

The pressures that are attained inside the combustion chamber are extremely high (~100 bar). Such a high pressure gradient (between chamber pressure and surrounding pressure), causes very high flow-rates. Under such conditions, the compressibility of the gas becomes important and must be considered while calculating the outlet flow rate. It is also expected that at such high flow rates, 'choking flow' would set in. Choking flow is a condition where the outlet flow velocity cannot have a Mach number (Ma) greater than 1 irrespective of the pressure gradient driving the flow. This would have significant influence on the system behavior and must also be considered formulating model. the Another consideration is the exit port geometry. It is necessary to calculate the frictional losses of the decomposition products while exiting to predict the outlet flow rate accurately.

The outlet flow in modeled as an adiabatic flow through a nozzle. This 'ideal' flow-model needs to be corrected accounting for the chamber pressure and the exit port configurations; a correction factor (ϕ) is multiplied to the ideal flow-rate. Through preliminary calculations, it was found the system operates under choking flow conditions for most of the period of operation. Under choking flow conditions, the

correction to the ideal model does not depend on the chamber pressure. Hence, the correction factor depends only on the exit port configuration or specifically on the frictional head loss of the flow through the exit port. The frictional head loss is given as:

$$N = \frac{4f_F L}{d} \tag{17}$$

Here, N is the effective frictional head loss. fr is the fanning friction factor and an equivalent L/d ratio is calculated for the given geometry. fr can be calculated from the Moody charts^[2]. f_F depends on the Reynolds number of the flow and the effective roughness of the exit port. However, at very high Reynolds numbers, as are obtained in this system, fF is constant and independent of the Reynolds number and depends only on the effective roughness of the pipe which is constant for a given geometry. Hence, for this system we can assume that fF is constant. Also for a fixed geometry, we can assume that the equivalent L/d ratio is constant. Hence, we can assume that the frictional head loss for the system is constant. As the correction factor under choking flow conditions depends only upon the frictional head loss, we can assume that the correction factor is constant for the system. The gas flux for the ideal compressible flow through an adiabatic nozzle is given by:

$$\dot{G}_{exit} = P_g (Ma) \left[\frac{g_c (mw) k_g}{R T_g Y^{(k_g + 1)/(k_g - 1)}} \right]^{1/2}$$
 (18)

where

$$Y = \left[\frac{P_g}{P_{ambient}}\right]^{\binom{k_g-1}{k_g}} Ma = \sqrt{\frac{2(Y-1)}{\binom{k_g-1}{k_g-1}}}$$
(19)

However, as Ma = 1, conditions for choking flow are obtained. The Ma number cannot increase beyond this value and is kept constant at 1 for as long as the coking flow conditions persist. Y is calculated by taking Mach number equal to 1.

The rate of outlet flow is thus given by:

$$\dot{m}_{exit} = \Phi \dot{G}_{exit} A_{orifice} \text{ (kg/s)}$$
 (20)

The model equations (4), (5), (9), (10), (13), (14) & (15) form a family of simultaneous ordinary differential equations. The equations were found to be stiff during preliminary simulations. Hence, an implicit method is

required to solve this system. The SUNDIALS suite of packages (https://computation.llnl.gov/casc/sundials/), in particular the CVODE solver was used in the present work. The method selected in this work was variable-order, variable-step multi step methods based on backward difference formula and Newton type of iterations. Adequate care was taken to ensure that selected solver settings and tolerances have no influence on simulated results. Some of the simulated results are discussed in the following section.

The initial conditions for the system are shown in Table 1. The composition of the combustion gas was calculated using a chemical equilibrium analysis code (NASA SP-273), specifically for the solid propellant used. The combustion gas is treated as an ideal mixture of the species in proportions obtained from the chemical equilibrium analysis code. The combustion chamber was assumed to be initially filled with the combustion gas.

Results and Discussion

Preliminary simulations were carried understand the key issues, sensitive parameters and overall reasonableness of the predicted results. Typical values of parameters used for these simulations are listed in Table 1. The typical simulated pressure profile and temperature profile of the combustion chamber are shown in Figure 2. In the present study the units of pressure will be scaled using an arbitrary factor. The simulated pressure profile is mainly characterized by the four key parameters: the location of maximum pressure (denoted by tmax, Pmax) and the location of complete burnout (denoted by tburn, Pburn). The characteristic shape of the pressure profile is a direct influence of the change in size and shape of the solid propellant brick.

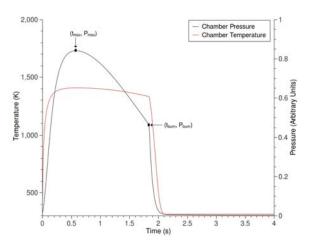


FIG. 2 TYPICAL SIMULATION PROFILE

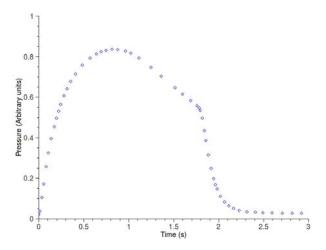


FIG. 3 TYPICAL EXPERIMENTAL PROFILE

Typical experimental data obtained in earlier studies (NSTL Internal Report) is shown in Figure 3 in arbitrary units of pressure. It can be seen that the presented model was able to capture the observed pressure profiles in the combustion chamber reasonably well. In order to obtain quantitative agreement with the experimental data, it is necessary to identify the mathematical model with the considered experimental system. Unfortunately the experimental system was not equipped for measuring the chamber temperatures. Therefore the model identification or calibration had to be carried out by relying only on the observed pressure profiles in the chamber. The kinetics of the used solid propellant (i.e. the burning rate function) was also not available from the independent measurements. It was therefore necessary to obtain the key kinetics parameters of the solid propellant also from the observed pressure profiles. Considering these several limitations of the available information about the experiments and large number of parameters used even in the simple model presented here, the development of a systematic methodology for calibrating the model is required. Such an attempt was made in this work which is discussed in the following.

The specific parameters which are required for carrying out simulations, but are not available are, the heat of decomposition of solid propellant (ΔH), the effective heat transfer coefficients (chamber side, heff,ch and solid propellant side heff,sp), the burning rate law parameters (a and n) and the outlet flow rate correction factor (ϕ). Although these parameters are not known, they are specific to our system and are constants. Hence, we obtain the unknown values by matching simulation results to the experimental results for a particular experimental configuration.

This 'calibrated' model will then used to simulate for other experimental configurations and the simulation results will be compared with experimental results. This would establish the usefulness of our model.

TABLE 1 BASE CASE PARAMETERS USED FOR SOLID PROPELLANT DECOMPOSITION

Initial Conditions		
Solid Propellant Temperature (K)	300.0	
Chamber Wall Temperature (K)	300.0	
Pressure within combustion chamber (Pa)	101325.0	
Combustion Products Temperature (K)	300.0	
Solid Propellant Configuration		
Inner Diameter (in m)	0.035	
Outer Diameter before inhibition (in m)	0.067	
Length (in m)	0.085	
Density (kg/m³)	1610	
Mass (before inhibition) (kg)	0.349	
Combustion chamber configuration		
Volume of Combustion chamber (m³)	7.50E-04	
Area of Exit Orifice (m²)	2.83E-05	
Heat Exchanger Configuration		
Flow rate of coolant (kg/s)	1.0	
Specific Heat of coolant (J/(kg-K))	4186	
Density of coolant (kg/m³)	1000	
Inlet Temperature of coolant (K)	300.0	
Outlet Temperature of coolant (K)	373.0	
Ambient Conditions		
Ambient Temperature (K)	300.0	
Ambient Pressure (Pa)	101325.0	

Calibrating a model which has five parameters is a complex problem. To simplify the problem, we decided to investigate if some parameters among the unknown are 'equivalent' for the purposes of calibrating the model. First we need to elaborate as to what equivalence of parameters means. Consider a

pair of parameters. All other parameters remaining constant, if the change in the pressure and temperature profiles caused by altering one parameter can be nullified by adjusting the other parameter suitably, they are 'equivalent' from the stand point of calibrating the model. Hence, for such a pair, we can assign a *fixed* representative value to one parameter and use the other parameter to calibrate the model. Thus, the number of parameters required to calibrate the model are reduced. Such an investigation was carried out for our system, in an attempt to reduce the number of parameters and hence, the complexity of the calibration process.

It was found that the heat of decomposition (ΔH) and the effective coefficients of heat transfer to solid propellant brick (heff,sp) as well as chamber wall (heff,ch) are equivalent when comparing the pressure profiles (Figure 4). The heat of decomposition is also shown to be equivalent to the outlet flow correction factor (ϕ) (Figure 5). Hence, all the four aforementioned parameters are equivalent with regards to the pressure profile. However, the resulting temperature profiles for these equivalent pressure profiles were different. In simulations, the temperature was seen to attain a 'steady state' value after a brief initial build up period (Figure 2). Unfortunately, detailed experimental results regarding the temperature profile could not be obtained. However, the existence of such a 'steady state' temperature was verified experimentally and its value was known (~1500 K). It was observed that the temperature profile was influenced only by the ΔH , heff,sp, and heff,sp parameters. The change in the aforementioned parameters only resulted in a change in the steady state value of the temperature. The temperature profile was found to be independent of ϕ . As all the four parameters are equivalent with regards to pressure, it is sufficient to choose only one of these parameters for the purpose of calibration using the pressure profiles. It is desirable to choose ϕ as a calibration parameter among the four parameters as it does not influence the temperature profile. Moreover, as the 'steady state' temperature is not expected to change for our system, ΔH , heff,sp, and heff,sp are assigned suitable representative values such that the steady state temperature attains a value of around 1500 K. It was found that the burning rate law coefficient (a) and the burning rate law exponent (n) were equivalent. This equivalence is demonstrated in Figure 6. The equivalence is within the error of the performed experiment. The temperature profiles showed adequate agreement but are not shown here. Hence,

the parameters ' ϕ ' and 'a' are the only required parameters for calibrating our model.

We will now proceed to calibrate the model using a particular experimental configuration. The calibrated model will then be used to simulate for the rest of the experimental configurations. The characteristic points for a typical pressure curve are (tmax, Pmax) and (tburn, Pburn) shown in Figure 2. For calibrating our model we will match these characteristic points obtained from simulations to the corresponding points of the experimental result for a particular solid propellant configuration. However, with only two independent parameters, we can match only two out of the four characteristic values. We do not have control over the other two. We need to choose two characteristics for this purpose. Out of all the possible pairings, it was decided to choose Pmax and tourn as they are critical characteristics in terms of design of a combustion chamber. Test Case 210 (NSTL Internal Report) was chosen to be used for calibrating the model. Figure 7 shows the comparison between the simulated results and the experiments. The characteristics P_{max} and t_{burn} have been matched adequately. The configuration of the solid propellant is 71x35x80 (D_oxD_ixL in mm).

The obtained values of the parameters (ϕ_0 , a_0) will now be used to simulate for other solid propellant configurations. The simulated pressure profiles will then be compared with the corresponding experimental profiles. Figure 8 shows a comparison for the case of Test Case 207 (NSTL Internal Report). The configuration of the solid propellant for this case is $67 \times 30 \times 100$. As can be seen the calibrated model results agree reasonably well with the experiments.

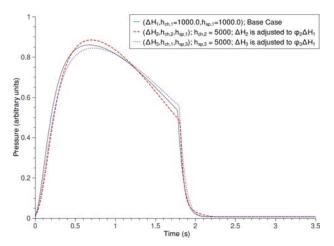


FIG. 4 EQUIVALENCE OF HEAT OF DECOMPOSITION TO THE CHAMBER SIDE EFFECTIVE HEAT TRANSFER COEFFICIENT AND SOLID PROPELLANT SIDE HEAT TRANSFER COEFFICIENT

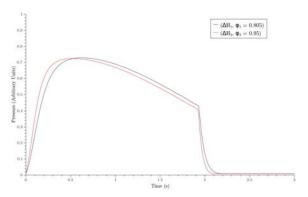


FIG. 5 EQUIVALENCE OF HEAT OF DECOMPOSITION TO THE OUTLET FLOW CORRECTION FACTOR

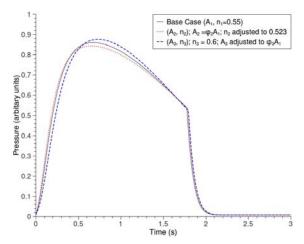


FIG. 6 EQUIVALENCE OF BURNING LAW EXPONENT AND BURNING LAW COEFFICIENT

The calibrated model parameters can be further fine tuned to obtain a better match with regards to the chosen characteristics. The required change in parameters is not significant. Figure 9 shows to comparison for the case of Test Case 208 (NSTL Internal Report). The configuration of the solid propellant for this case is 69x35x75. The comparison using the calibrated model is good, the value for tourn is predicted adequately however, the value of Pmax is over-predicted. The comparison of the overall profile looks good. After fine tuning the model by changing the model parameters slightly, better agreement with respect to P_{max} can be achieved, however, comparison of the overall profile is not as good as that using original parameters. Figure 10 shows a comparison for the case of Test Case 209 (NSTL Internal Report). The configuration of the solid propellant for this case is 67x35x85. The calibrated model is successful in predicting tourn however, it under predicts Pmax. Again the model can be fine tuned by changing the model parameters slightly so that there is a better agreement between the simulated and experimental characteristics. Thus, the present model is reasonably successful at predicting the burning profile of a solid propellant in a combustion chamber.

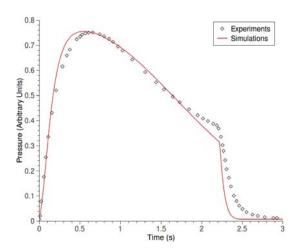


FIG. 7 COMPARISON BETWEEN EXPERIMENTAL AND SIMULATED PRESSURE PROFILE AFTER CALIBRATING MODEL FOR TEST CASE 210 (NSTL INTERNAL REPORT)

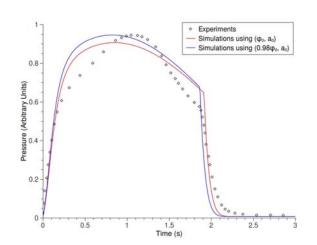


FIG. 8 COMPARISON BETWEEN EXPERIMENTAL AND SIMULATED PRESSURE PROFILE FOR TEST CASE 207 (NSTL INTERNAL REPORT)

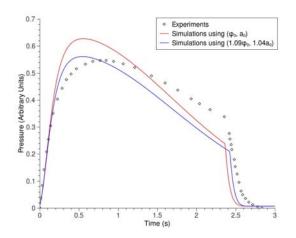


FIG. 9 COMPARISON BETWEEN EXPERIMENTAL AND SIMULATED PRESSURE PROFILE FOR TEST CASE 208 (NSTL INTERNAL REPORT)

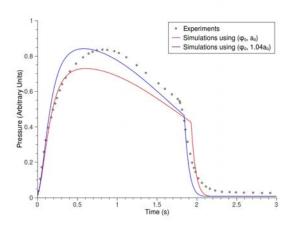


FIG. 10 COMPARISON BETWEEN EXPERIMENTAL AND SIMULATED PRESSURE PROFILE FOR TEST CASE 209 (NSTL INTERNAL REPORT)

The model can be used to study the influence of shape of the solid propellant brick on the combustion profile. To demonstrate this, simulations were carried out for three different configurations of the brick (SP1 (67x35x85), SP2 (69.76x40x85), SP3 (64.55x30x85)), however, all having the same quantity of solid propellant. As can be seen from Figure 11, the shape of the brick has a significant impact on the combustion profile. The model can be used to study the influence of shape of the solid propellant brick on the combustion profile. To demonstrate this, simulations were carried out for three different configurations of the brick (SP1 (67x35x85), SP2 (69.76x40x85), SP3 (64.55x30x85)), however, all having the same quantity of solid propellant. As can be seen from Figure 11, the shape of the brick has a significant impact on the combustion profile.

The shape of the internal bore of solid propellant brick is also an important design consideration. Different internal shapes have been known to be used for different design purposes e.g. a star-shaped internal bore is used for a relatively constant pressure profile (as the total burning surface area remains nearly constant as burning progresses). The internal shape or size of the bore may be changed keeping the desired pressure profile in mind. The present model is a useful tool to evaluate such design considerations of the solid propellant.

In many propulsion applications, solid propellants are used as an igniter for liquid propellants. The model presented here can be extended to simulate the behavior of such propulsion systems where liquid propellant is injected after igniting solid propellant (with some delay). Such an attempt at extending the model has been made and would be a part of a separate note.

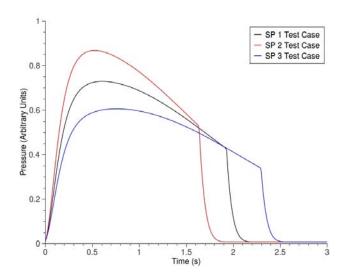


FIG. 11 INFLUENCE OF SOLID PROPELLANT SIZE ON THE BURNING PROFILE

In the present model we have neglected the influence of erosive burning. Erosive burning is prominent typically in internal burning solid propellant bricks like the one encountered in the present study. Also, the detailed kinetics of the burning process of the solid propellant brick has not been considered. Instead an empirical correlation was used to predict the burning rates. The aim of the present study was to identify the key parameters governing the system and reasonably predict their influence on macroscopic system parameters. Obtaining reasonable estimates for crucial system parameters macroscopic (pressure, temperature) especially in such a rapid, high temperature, high pressure system would serve in enhancing the experimental program. The present model is sufficient to reasonably explain the characteristic shape of the pressure profile as well as to obtain reasonable estimates of key parameters like maximum pressure and the total burning time. Hence, for the purpose of the present study, keeping our aim in mind, we decided to retain the simplicity of the model and make the aforementioned assumptions. The present model may easily be refined further to include the effects of erosive burning or detailed kinetics on the system parameters if such a need arises.

Conclusions

In this study, we have presented a simple model for the solid propellant decomposition in a combustion chamber with an exit port. The model predictions are compared with the experimental data obtained in earlier studies. Appropriate methodology for calibrating the model parameters is developed and its usefulness was demonstrated by comparing the simulated pressure profiles with the experimental pressure profiles. The model is then used to investigate influence of size and shape of the solid-propellant brick on generated pressure profiles. Some of the key conclusions of the present study are as follows:

- The characteristic pressure profile of a combustion chamber during the decomposition of the solid propellant can be simulated by appropriate selection of effective burning law coefficient (a) and the correction factor (ϕ) for the specific geometry of the combustion chamber.
- The appropriate values of fitted parameters, burning law coefficient (a) and the correction factor (φ) can obtained by matching the simulated values of maximum pressure (P_{max}) and time required for complete burning (t_{burn}) with the corresponding characteristic points of the experimental data for a given configuration.
- The shape and size of the solid propellant has a significant influence on the burning profile. The present model can be used to evaluate the design of solid propellant brick in such respects.

The presented model and results will be useful for selecting appropriate solid propellants for propulsion systems and for the development of a system level model of propulsion systems.

Notations

\dot{m}_{burn}	Rate of Decomposition of Solid propellant
\dot{m}_{exit}	Rate of Outlet Flow of Decomposition Products
\dot{S}	Rate of Regression of Solid Propellant Surface
A_b	Area of Burning
a	Burning Law Coefficient
n	Burning Law Exponent
C_p	Specific Heat Capacity
V	Volume
T	Temperature

P	Pressure
h	Enthalpy
ho	Density
ΔH	Heat of Decomposition
m	Mass
MW	Molecular weight
$Y_{m,sp}$	Mass Fraction of Species 'm' in the
	Decomposition Products of Solid Propellant
Y_m	Mass Fraction of a Species 'm'
L	Length of Solid Propellant Brick
D_i	Inner Diameter of Solid Propellant Brick
D_o	(Uninhibited) Outer Diameter of Solid
	Propellant Brick
$h_{\it eff,ch}$	Effective Heat Transfer Coefficient for Heat
	Transfer from Decomposition Products to Combustion Chamber
$h_{\it eff,sp}$	Effective Heat Transfer Coefficient for Heat
	Transfer from Decomposition Products to Solid Propellant
$\dot{Q}_{\scriptscriptstyle HX}$	Rate of Heat Loss to the Heat Exchanger
k	Compressibility
R	Universal Gas Constant
\dot{G}_{exit}	Gas flux at Exit Port
Φ	Correction Factor for Calculating Gas Exit Flow-rate
Subscripts	
8	Combustion Gases
sp	Solid Propellant

ch Combustion Chamber

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